with a value of 17.804 MeV obtained by Green by fitting nuclear masses.[7] Thus, our calculation leads to a surface thickness and surface energy which are both in reasonable agreement with experimental data.

[7] A. E. S. Green, Phys. Rev. **95**, 1006 (1954).

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Angular Correlations of Cascade Gamma Rays in the Decay of Lu^{172}

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Angular correlation experiments have been performed on the 1095–78.7, 901–1095, 901–78.7, and 1586–78.7 keV γ-ray cascades in Lu^{172}. The results of the directional correlation and polarization correlation measurements on the 1095–78.7 keV gamma rays establish the spin and parity of the 1174-keV excited state as 3+. The other measurements are consistent with previous assignments for the spins and parities of the 1664- and 2075-keV excited states.

INTRODUCTION

THE decay of Lu^{172} has been studied by Wilson and Pool[1] using γ-γ coincidence techniques to determine the order and energies of the electromagnetic transitions. The spin and parity assignments of the various energy levels were based on the apparent presence of rotational band spectra and on the consistency of the transitions between different bands with the appropriate selection rules. The decay scheme was further expanded by Harmatz *et al.*[2] using conversion electron measurements with a magnetic spectrometer. A decay scheme showing some of the essential features determined by these two studies is presented in Fig. 1.

Since the 1174-keV level appears to be the ground state of a rotational band with $K=3$, a spin of 3 was assigned to this level. The branching ratios and intensities were the basis for a positive parity assignment. In addition the 1664-keV and the 2075-keV level were assigned spins of 3 and 4, respectively.

In this work, angular correlation experiments have been performed in order to provide a definite assignment for the spin and parity of the 1174-keV level and to indicate probable assignments for the 1664- and 2075-keV levels.

EXPERIMENTAL PROCEDURE

Lu^{172} was produced by a 2-h irradiation of an enriched sample of Yb_2O_3 with 12-MeV protons in the Oak Ridge National Laboratory 86-in. cyclotron. Lu^{172} has a half-life of 6.7 days and decays by electron capture to levels in Yb^{172}. The only other activity observed to be present in the source was a small amount of Lu^{171} whose γ rays do not interfere with the measurements carried out in this study. It was found that the Lu activity was readily dissolved in HCl while the Yb_2O_3 was relatively insoluble. Therefore, no additional chemical separation was performed.

The mean lifetime of the 78.7-keV level has been

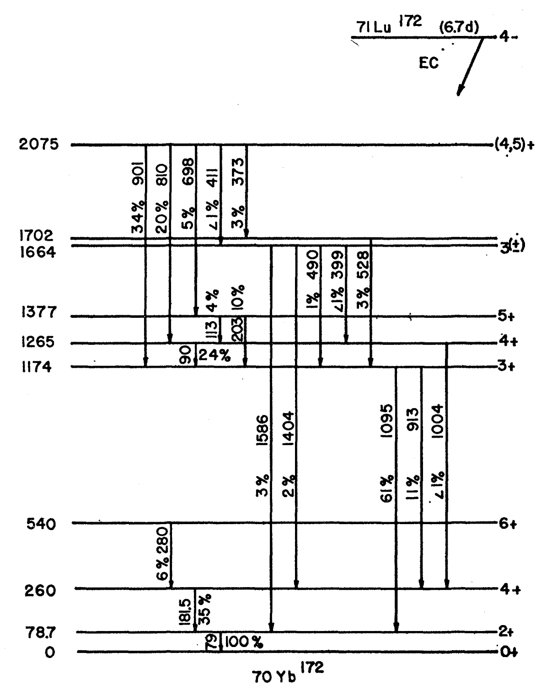

FIG. 1. Decay scheme of Lu^{172}.

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[1] R. G. Wilson and M. L. Pool, Phys. Rev. **118**, 1067 (1960).

[2] B. Harmatz, T. H. Handley, and J. W. Mihelich, Phys. Rev. **123**, 1758 (1961).

found by Birk *et al.*[3] to be 2.4±0.2 nsec. This lifetime is sufficiently long that attenuation of the angular correlation may occur. To minimize this effect, the source was used in the form of a dilute HCl solution. The source was placed in a cylindrical glass tube having a 0.030-in. wall thickness and an inside diameter of $\frac{3}{16}$ in. This type of source holder was used in both the directional correlation and the polarization correlation experiments.

The experimental apparatus for the directional correlation experiments consisted of three scintillation counters composed of 1-in.-diam×2-in.-long Harshaw NaI(Tl) crystals mounted on EMI 9536B photomultiplier tubes. The resolution of the counters was approximately 8% at an energy of 662 keV. The coincidence circuitry was of the standard fast-slow type having a resolving time of 30 nsec. One counter was kept fixed, and the other two were permuted successively through positions at 90°, 135°, and 180° with respect to the fixed counter; in order to reduce the effects of drifts in the detectors and associated circuitry, these permutations were made at regular intervals.

The single-channel pulse-height analyzer for the fixed counter was set to accept only one of the gamma rays. The coincidence and singles spectra from each of the movable counters was recorded with a 400-channel analyzer using a subgroup programmer circuit.[4] This circuit routes the singles and coincidence spectra of each movable counter into different 100-channel subgroups; the signal paths followed by these pulses are in all other respects identical. Thus, it is possible to normalize the coincidence counting rates by the simultaneously measured singles counting rates and also to determine the interference to the measured correlation caused by other radiations from the source. A typical set of singles and coincidence spectra as recorded in this manner is shown in Fig. 2. The channels selected for analysis are indicated by the crossbars.

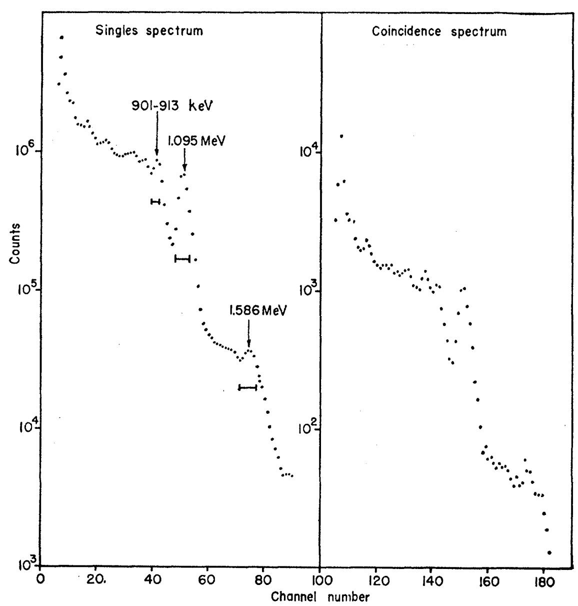

FIG. 2. Singles and coincidence spectra as measured using the subgroup programmer. The horizontal crossbars represent channels used in the angular correlation analysis.

DIRECTIONAL CORRELATION RESULTS

In order to determine the spin of the 1174-keV level, directional correlation measurements were made on the 1095–78.7 keV γ-ray cascade. Solid angle corrections were applied to the Legendre polynomial coefficients of the measured correlation using the method of Rose[5]; corrections were also made for the interference of the x ray and the 91-keV γ ray with the 78.7-keV gamma ray. To make the latter correction, coincidence spectra in the fixed counter were measured with the pulse-height discriminator of one of the movable counters set to accept only the 1095-keV γ ray. Such a coincidence spectrum is shown in Fig. 3. The 88-keV γ ray of Cd^{109} was measured with the fixed counter and this spectrum was used as a basis for determining the relative amounts of the x ray, the 91-keV γ ray, and the 78.7-keV γ ray in the discriminator window. The intensities of the x ray and the 91-keV γ ray relative to the 78.7-keV γ ray were 30 and 5%, respectively, in this window. The directional correlation of the x ray with the 1095-keV γ ray was taken to be isotropic. The conversion electron data of Harmatz *et al.*[2] indicates that $E2/M1=3$ for the 91-keV γ ray, and this value was used to calculate possible angular correlation coefficients for the 91–1095

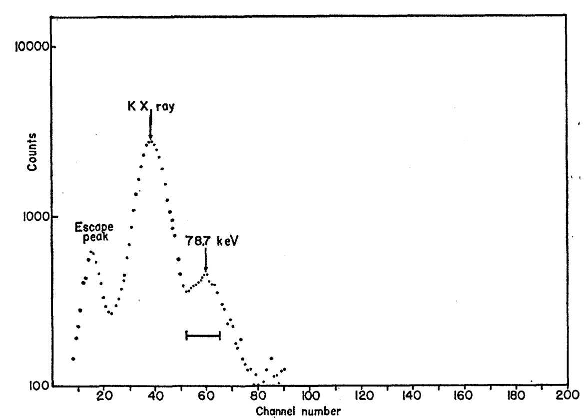

FIG. 3. Coincidence spectrum recorded in the fixed counter when one of the movable counters was set on the 1095-keV γ ray. The horizontal crossbar indicates the window used in the fixed counter in the angular correlation measurements.

[3] M. Birk, A. E. Blaugrund, G. Goldring, E. Z. Skurnik, and J. S. Sokolowski, Phys. Rev. **126**, 726 (1962).

[4] E. Brooks Shera and Karl J. Casper, Nucl. Instr. Methods **17**, 174 (1962).

[5] M. E. Rose, Phys. Rev. **91**, 610 (1953).

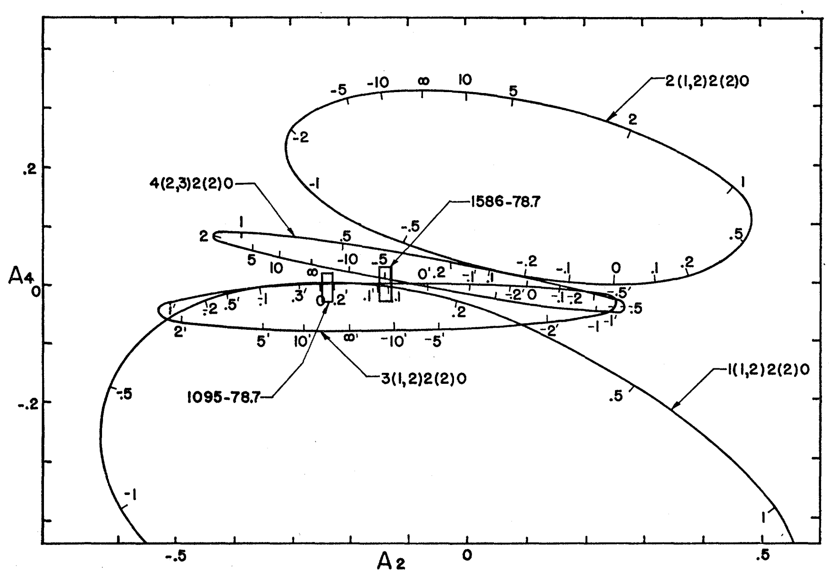

FIG. 4. Parametric representation of the Legendre polynomial coefficients for the directional correlation experiments with experimental points.

keV γ-ray cascade. If all probable spin sequences are considered, the net effect of the interference of the 91-keV γ ray is to add a maximum uncertainty of ± 0.025 to the A_2 coefficient and a negligible uncertainty (compared to the statistical error) to the A_4 coefficient. The final corrected values of the polynomial expansion coefficients determined by the present experiments are shown in Table I, along with the results of the other directional correlation measurements made in this work.

A parametric representation[6] of the Legendre polynomial coefficients as a function of the mixing ratio is shown in Fig. 4, together with the experimental points. The results for the 1095–78.7 keV cascade are seen to be consistent with the following assignments:

$$3(1,2)2(2)0 \quad \delta=0.225,$$
$$1(1,2)2(2)0 \quad \delta=0.007,$$

where δ is the ratio of the reduced matrix element for the emission of $(L+1)$ to that of L radiation. The 4(2,3)2(2)0 spin sequence can be eliminated, since the values are substantially outside the statistical error and since such an assignment would involve a large contribution from octupole radiation. Such a contribution is not consistent with the observed intensity of the transition.

TABLE I. Directional correlation results.

Cascade (keV)	A_2	A_4
1095–901	$+0.0311\pm0.0220$	-0.0200 ± 0.0435
901– 78.7	$+0.0825\pm0.0136$	-0.0303 ± 0.0278
1095– 78.7	-0.2397 ± 0.0323	-0.0060 ± 0.0241
1586– 78.7	-0.1303 ± 0.0151	$+0.0015\pm0.0318$

Further discrimination between the two possibilities in the 1095–78.7 cascade may be obtained by considering the triple cascade as shown in Fig. 5. Reich *et al.*[7] have pointed out that it is possible to obtain information about the spin j_2 independent of the spin j_1 and the multipolarity of the first transition. This information is in addition to that derived from the usual second-third $(j_2\rightarrow j_3\rightarrow j_4)$ angular correlation and may or may not be redundant.

For the first-third $(j_1\rightarrow j_2,\ j_3\rightarrow j_4)$ directional correlation the second Legendre polynomial coefficient is

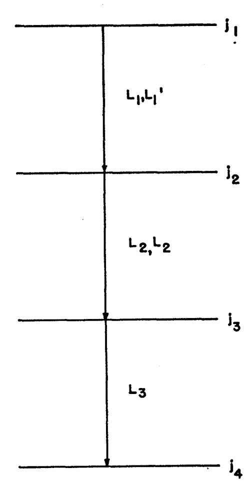

FIG. 5. Triple γ-ray cascade with upper two transitions of mixed multipole order L, L' where $L'=L+1$.

[6] P. S. Jastram, G. T. Wood, and J. P. Hurley, Bull. Am. Phys. Soc. 3, 65 (1958); see also, M. K. Ramaswamy and P. S. Jastram, Nucl. Phys. 16, 113 (1960).

[7] C. W. Reich, R. P. Schuman, J. R. Berreth, M. K. Brice, and R. L. Heath, Phys. Rev. 127, 192 (1962).

given by

$$A_2(1\text{-}3)=[(1+\delta_1^2)(1+\delta_2^2)]^{-1}[F_2(L_1L_1j_1j_2)+2\delta_1F_2(L_1L_1'j_1j_2)+\delta_1^2F_2(L_1'L_1'j_1j_2)]$$
$$\times(-1)^{L_2-j_2-j_3}[(2j_2+1)(2j_3+1)]^{1/2}[W(j_2j_2j_3j_3;2L_2)-\delta_2^2W(j_2j_2j_3j_3;2L_2')]F_2(L_3L_3j_4j_3).$$

For the directional correlation between the first and second transitions ($j_1\rightarrow j_2\rightarrow j_3$) the second Legendre polynomial coefficient is

$$A_2(1\text{-}2)=[(1+\delta_1^2)(1+\delta_2^2)]^{-1}[F_2(L_1L_1j_1j_2)+2\delta_1F_2(L_1L_1'j_1j_2)+\delta_1^2F_2(L_1'L_1'j_1j_2)]$$
$$\times[F_2(L_2L_2j_3j_2)+2\delta_2F_2(L_2L_2'j_3j_2)+\delta_2^2F_2(L_2'L_2'j_3j_2)].$$

The ratio of these coefficients is

$$\frac{A_2(1\text{-}2)}{A_2(1\text{-}3)}=\frac{[F_2(L_2L_2j_3j_2)+2\delta_2F_2(L_2L_2'j_3j_2)+\delta_2^2F_2(L_2'L_2'j_3j_2)]}{(-1)^{L_2-j_2-j_3}[(2j_2+1)(2j_3+1)]^{1/2}[W(j_2j_2j_3j_3;2L_2)-\delta_2^2W(j_2j_2j_3j_3;2L_2')]F_2(L_3L_3j_4j_3)}.$$

In this analysis, it is assumed that $\delta_3=0$. A graph of this ratio as a function of the mixing ratio δ_2 is shown in Fig. 6.

The results of the directional correlation experiments on the 901–78.7 keV cascade and the 901–1095 keV cascade are given in Table I. The ratio of the A_2 coefficients from these measurements is

$$\{A_2(1\text{-}2)\}/\{A_2(1\text{-}3)\}=+0.377\pm0.274.$$

The value for this ratio is plotted in Fig. 6. Before interpreting this result, some consideration must be given to the problem of interferences among competing radiations in these measurements.

The pulse-height discriminator for the fixed counter was set to accept the 78.7-keV γ ray for the 901–78.7 keV cascade correlation measurement and the 901-keV γ ray for the 901–1095 keV cascade correlation measurement. Since the 1095-keV γ ray and the 913-keV γ ray are not in coincidence, there was no problem with interference between the 901- and 913-keV γ rays in the measurement of the 901–1095 keV directional correlation. However, some interference from the 913-keV γ ray is to be expected in the 901–78.7 keV directional correlation. Since this interference cannot be eliminated, it can

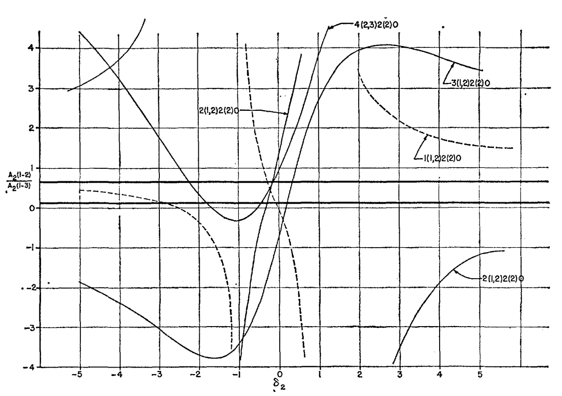

FIG. 6. Ratio of A_2 coefficients for a first-second directional correlation to those for a first-third directional correlation plotted as a function of the mixing ratio for the second (intermediate) transition. The experimental result is presented here as the area between the two heavy horizontal bars.

only be said that the preceding values for the directional correlation coefficients and the ratio of the A_2 coefficients are not inconsistent with the assignment of spin 3 to the 1174-keV level.

An attempt was made to determine the spin of the 2075-keV level by analyzing the A_2 and A_4 coefficients

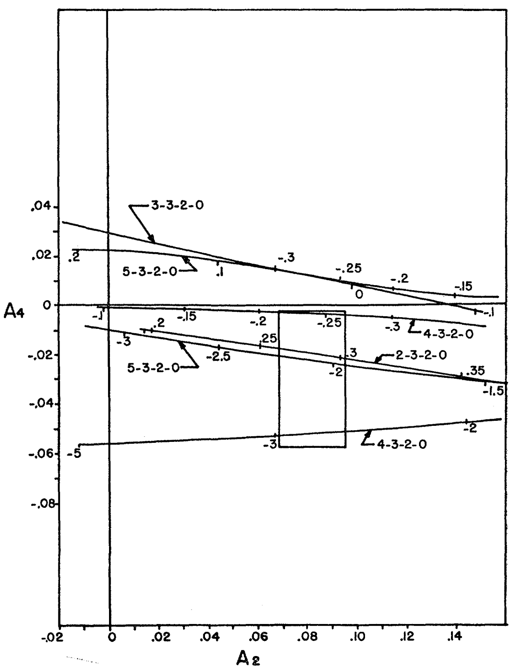

FIG. 7. Parametric representation of A_2 and A_4 coefficients for a first-third directional correlation with the experimental value for 901–78.7 cascade shown. The spin of the 1174-keV excited state is taken to be 3, and the mixing ratio for the 1095-keV transition is taken to be 0.225.

for the first-third correlation. A graphical representation is shown in Fig. 7 with an assumed spin of 3 for the 1174-keV level and a mixing ratio of 0.225 for the 1095-keV γ ray. The curve for the 1-3-2-0 spin sequence falls outside the limits of the graph. Other graphs were prepared, one for the first-second correlation and one for the ratio of the A_2 coefficients for the first-third and second-third correlations. However, no additional information was obtained and the only spin assignment that could be definitely excluded was that of 1 for the 2075-keV level. The results displayed in Fig. 6 and the results for the other correlations were consistent with all other spin assignments.

POLARIZATION DIRECTION CORRELATION EXPERIMENT

In order to make a definite choice between the two possible spin assignments for the 1174-keV level, an experimental measurement of the linear polarization of the 1095-keV γ ray was made. A polarization-sensitive apparatus utilizing Compton scattering was constructed

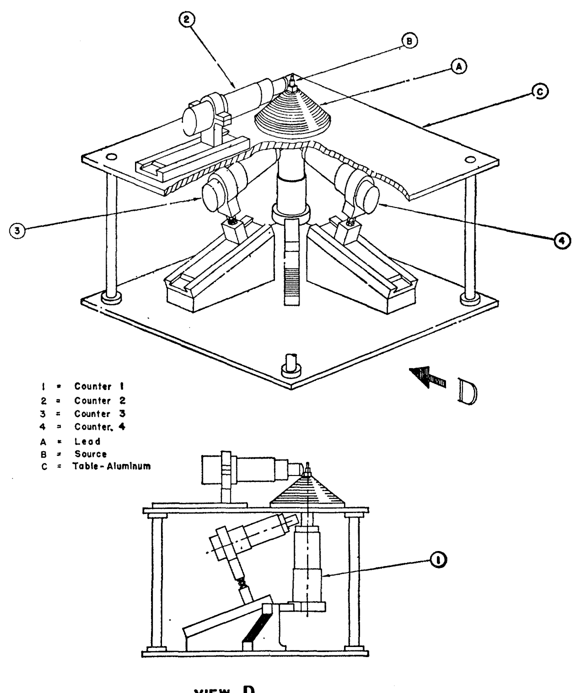

FIG. 8. Schematic drawing of the polarization correlation apparatus.

and is presented schematically in Fig. 8. The vertical scattering counter used a 1-in.-diam by 2-in.-long plastic phosphor, while the other counters were similar to those used in the directional correlation experiments. The conical lead shield was 4 in. high with a central $\frac{3}{8}$-in. collimating hole.

The pulses from the vertical counter and each side counter (3 or 4) were summed, and the pulse-height discriminators in the two sum channels were set to span the 1095-keV sum peak. The top counter was set with a narrow window on the 78.7-keV γ ray. The 1-2-3, 1-2-4, 1-3, and 1-4 coincidence rates were recorded, and the triple coincidence rates were then normalized by the double coincidence rates. Counters 3 and 4 were alter-

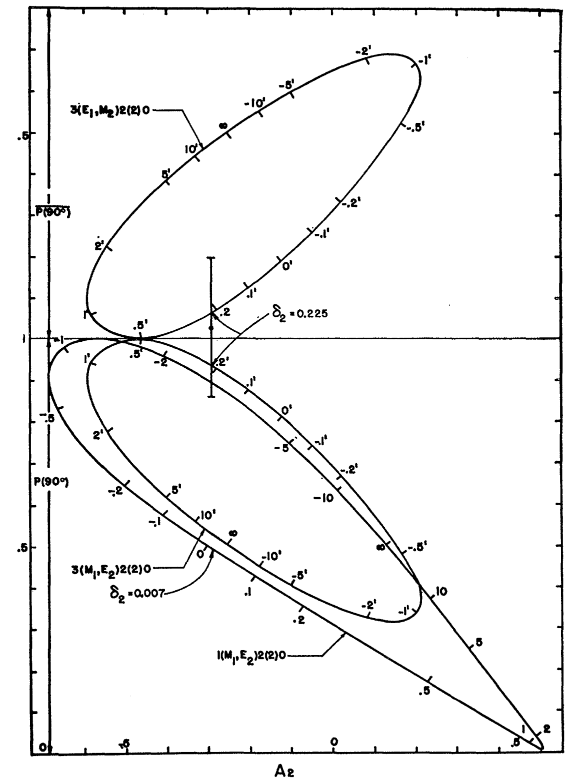

FIG. 9. Parametric representation of the polarization correlation with experimental value for the 1095–78.7 keV cascade shown. The values of δ_2 consistent with the directional correlation results are indicated.

nated between the positions parallel and perpendicular to the plane of counters 1 and 2.

The polarization p depends upon the polarization sensitivity R of the polarization apparatus through the relation[8]

$$p=(R-n)/(Rn-1),$$

where n is the ratio of the counting rate parallel to the plane of the counters 1 and 2 to the rate perpendicular to this plane.

Two methods were used to evaluate the sensitivity R. Using monoenergetic γ-ray sources which were placed in the normal source position and requiring that the sum of the pulses from the vertical counter and one of the side counters (3 or 4) equal to the total γ-ray energy, the spectra from the side counter were analyzed to determine the probability of scattering as a function of scattering angle. This was then used as a weighting function in the theoretical expression for the polarization sensitivity to obtain a value for R. Measurements on the linear polarization of the γ rays of Co^{60} were also made with the polarization apparatus. Since these transitions are pure electric quadrupole, the polarization with counters 1 and 2 perpendicular is known to be 1.40.

[8] F. R. Metzger and M. Deutsch, Phys. Rev. 78, 551 (1950).

This value of p, together with the experimentally determined value for n, gives a polarization sensitivity, extrapolated to 1.095 MeV, of 1.49 ± 0.31. This is in agreement with the value obtained from the first method of 1.40 ± 0.05.

The polarization of the 1095-keV γ ray as measured in this experiment is 1.035 ± 0.154. This value is plotted in Fig. 9 using the A_2 coefficient determined in the directional correlation experiment. The polarization correlation experiment, therefore, eliminates the possibility of a 1-2-0 spin sequence for this cascade. Since the directional correlation measurements exclude the 4-2-0 and 2-2-0 spin sequences, the curves for these assignments are not shown. In this representation, the sequence $1(E1,M2)2(2)0$ is a mirror image of $1(M1,E2)2(2)0$ about the line $p=1$, as is shown for the $3(1,2)2(2)0$ curves.

DISCUSSION

The directional correlation and polarization direction correlation measurements definitely establish the spin of the 1174-keV level as 3. This experimental result supports the previous assumption that the 1174-keV level forms the ground state of a $K=3$ rotational band. It was not possible to determine the multipolarity of the transition from the polarization correlation experiment; however, the very high mixing ratio for the 1095-keV γ ray excludes the possibility of an $E1+M2$ mixture for this transition, indicating that the parity assignment for this level is positive.

Assignment of spins and parities to the 1664- and 2075-keV levels is more difficult. The directional correlation results for the transition from the 1664-keV level are consistent with an assignment of spin 1 or 3. Since no ground-state transitions have been observed from this level, a spin of 3 is assigned to this state. The mixing ratio as determined by the directional correlation measurements is 0.09 ± 0.01 which indicates that the transition is probably $M1+E2$, in which case the parity of the 1664-keV level is positive.

Since the 1586-keV transition is quite weak and the polarization sensitivity at this energy is low, no polarization correlation was attempted.

From the 901–1095 keV directional correlation data, it can be seen that no definite spin assignment can be made for the 2075-keV level. Previous investigators concluded that this level should have a high angular momentum since the transitions to the ground-state rotational band are weak. Neither a spin of 4 nor 5 is excluded by the directional correlation data. However, in the case of a 5-3-2-0 sequence, the most probable mixing ratio for the 901 keV transition is $0<\delta<0.05$ showing that this transition would be almost pure electric quadrupole. In the case of a spin assignment of 4-3-2-0, the mixing ratio is high, thereby excluding the possibility that this is an $E1$ transition. Thus, a parity change is excluded in the 901-keV transition and the parity of the 2075-keV level can be taken as positive.

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